リマ首都圏における住戸建築の変容ダイナミクス：セルオートマトン・アプローチ

Dynamics of house state consolidation in Lima Metropolitan area: a cellular automata approach

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Abstract: House consolidation is the progressive structural improvement of a house. The aim of this paper is to identify, describe and simulate the process of house consolidation. Houses located in informally developed blocks in Lima metropolitan area, are hypothesized to upgrade its state according to a local mechanism of perceived influence. A specifically designed Cellular Automata model based on actual data at the micro-scale of the block permits to track state transition process. An updating algorithm is devised to reproduce house consolidation process under experimental conditions where the access to this dynamic system is limited to only the construction state variable. A study area composed by three subdivided agricultural parcels has been used to characterize state transition process. Simulation allows to understand how the general model fits to different combinations of block size, composition and configurations observed in the empirical data. Limitations of this approach are rooted on the use of the single variable of state to replicate the whole spatial configuration and temporal trajectories. However, this study permits us to describe this complex system at the scale of the house, and to have access to the necessary information to replicate consolidation process, presenting a novel approach for studying the dynamics of informal urbanization from the bottom-up.

Keywords: *urbanization, house consolidation, micro-scale, cellular automata,* Lima
キーワード：都市化，家の統合，マイクロスケール，セルオートマトン，リマ首都圏

1. The case for a model on the process of house consolidation

Urban growth in large tracks of cities located in developing countries is characterized by informal urbanization and self-building processes. In Puente Piedra and Carabayllo districs, located in northern Lima metropolitan area, increasing population (Figure 1) and housing demand among the urban poor has been driving above-mentioned processes. In the northern fringe of the city, former agricultural land is being converted to residential purposes through subdivision of parcels. Land owners subdivide land and sell lots to householders who occupy it and start to self-build their houses into the new spatial pattern of housing blocks. Observed improvement of houses in Chillon valley (Figure 2) resembles the phenomena called house consolidation in areas where "low-income owners tend to build onto their dwelling environment gradually over a long period through a process called housing consolidation" [1].

To make it possible to understand above-mentioned process, it is important to devise an explanatory framework that permit us to replicate the process and to track the states along time and space. Structure and process are related elements in urban systems development [2)3)4]. Current investigation focuses on the relationship between structure and process and explore the dynamics of consolidation at the microscale of the block and its housing components. A study on the dynamics of consolidation process and its effects along time in the space of the block is suggested to provide insights on how the structure is being built at bottom level, and in turn how the changing urban structure is influencing the process as well. Studies of informal urbanization using computational models addresses the urban process at macro scale of city growth [5], or the growing structure at microscale driving the irregular morphology of settlements [6]; Batty (2008) suggest that micro-dynamic models clearly formulate a process-approach to the study of urban growth that provides "a coherent set of ideas about dynamics at the individual as well as collective level with clear links to the spatial morphologies generated" [7]. The contribution

Figure-1. Population growth in Carabayllo (CAR) and Puente Piedra (PP) districts
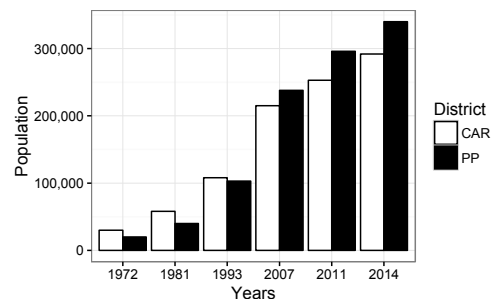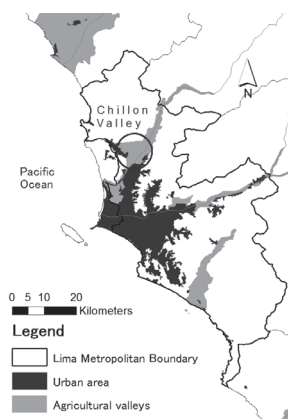

Figure-2. Location of Chillon valley

of current study is the inclusion of process oriented analysis at the micro-scale to make it possible to complement studies focused on patterns and morphologies; consolidation process has interesting spatial features

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such as decentralized growth and clearly defined spatial boundaries that can be modeled to make it possible to produce spatial structure (morphologies and patterns) as well as temporal trajectories. Expected outputs call for closing the observed gaps in representing consolidation process, linking the temporal transitions and the amount of development in the specific case of informal urbanization in Chillon valley. In a broad perspective, it addresses the dynamic processes of spatial organization at micro-scale and its development across time. Edmonds (2006) argues that physical space can be used as a proxy for social space [8]. Housing units are credited to be geographic units (occupying a space and possessing physical attributes), which have effects upon other similar units [9]. Subdivision is the actual process that creates new geographic units (blocks and houses) that are continuously upgrading its state while creating a independent and decentralized spatial pattern at the scale of the block. "To understand how collective order arises from seemingly random fluctuations, we must note how agents choose to interact with other agents and with the environment" [10]. Along the process of house consolidation, the operation in the system at block scale reflects individual decisions on house construction that leads to the upgrade of its state; thus, state transition (decision to upgrade) is driven by a rule that considers the specific characteristics of Chillon area during the period of study, specifically the transition rule that considers surrounding house conditions to specify the dynamic conditions prevailing in the informal urban system at block scale. Plausible interaction between neighbors is explicitly contemplated using a computational model. Thus, suggested approach at the bottom level recognize individual households whose decision to improve progressively its houses is hypothesized to respond interactions with close distance neighbors and resembles observed characteristics at block scale: independent self-building and membership of a block with clearly defined boundaries. Therefore, the resultant model should recognize individual differences and thus allow to illustrate the complex process of emergence of spatial and temporal patterns. The phenomena of heterogeneous and decentralized residential informal development, observed into the space of the block, is investigated through an empirical study using high-resolution imagery to retrieve data at the scale of the house; this research addresses the spatial and temporal dimensions of urban growth at the bottom level of houses as socio-physical units that are changing its construction states along the time into the boundary of a residential block. Consolidation is characterized by the independent trajectories of state transition that each house follows along the process. This feature makes it a dynamic problem that does not allow easy tracking along space and time. Spatial and temporal patterns of house consolidation are hypothesized to have been caused by a local mechanism that considers the influence of state conditions of surrounding houses. Solving this problem could make it possible for investors, planners, and policy-makers to provide explicitly-oriented services to the urban poor in a quantifiable and efficient way. At this point it is important to mention one important methodological problem that arises from observing, implementing and analyzing change along the time; the problem is that "the patterns obtained from remote sensing data usually represent the complex aggregate outcomes of many different individual processes, making it difficult to disentangle the effects of different variables and trends of interest" [11]. In addition, Lee (2002) observes that "in an urbanized environment, the geographic pattern of residential development is a complex phenomenon to model quantitatively" [12]. Recent work by Barros (2012), acknowledges the usefulness of complexity approach, mainly because the inherent features of independent and decentralized growth and resultant spatial heterogeneity that characteristics informal

development. She noted that the bulk of the housing stocks in Latin American cities now consists of upgraded (or in the process of upgrading) low-income residential areas, with many spontaneous settlements [5]. To make it possible to address the above-mentioned problem of describing and interpreting the complexity of informal development at bottom scale, Cellular Automata (CA) is going to provide the spatial-temporal setting for experiment and analysis. "Cells represent the basic units of spatial representation, which we assume are indivisible" [13]. "CA provides a computationally efficient technique for investigating the general nature of dynamical systems" [14]. "Growth patterns can emerge from the transition of states of individual cells" [15]. "One of the most potentially useful applications of cellular automata from the point of view of spatial planning is their use in simulations of urban growth at local and regional level" [16]. "In contrast to macroscopic land modeling, classical CA is a purely microscopic approach" [17]. CA models are thus credited to be capable to produce dynamic spatial models since their theoretical basis considers cell and neighborhood states, along with an explicit state transition rule. Complexity considers self-organization and path-dependence properties that drives the emergence of spatial and temporal patterns of transition in construction states at the scale of housing blocks, which are going to be tracked using both empirical data and simulation outputs. Observation of spatial transition patterns will provide information to build the model. This study then develops the argument that the subtle interaction and influence between neighbors, embedded into the improvements efforts in the whole block, causes the emergence of change in spatial and temporal dimensions of informal development into the space of the block. Critical in the notion of currently studied informal development is the bottom-up approach, which may explain the origins of the structure in the dynamic urban fabric of the informal urban fringe in northern Lima, where a large amount of population comprised by low-income households who cannot afford adequate housing and seek alternatives in cheap informally subdivided agricultural parcels, where they buy a clearly delimited lot that belongs to a structured block that lack basic services.

2. Building a model of consolidation process from empirical data
(1) Sampling and visual Identification of house states

The study area comprises four former agricultural parcels that are facing sub-division located along the urban fringe in Chillon valley. In the first selection made to retrieve data and information on the spatial characteristics of consolidation, three main criteria were used to select the

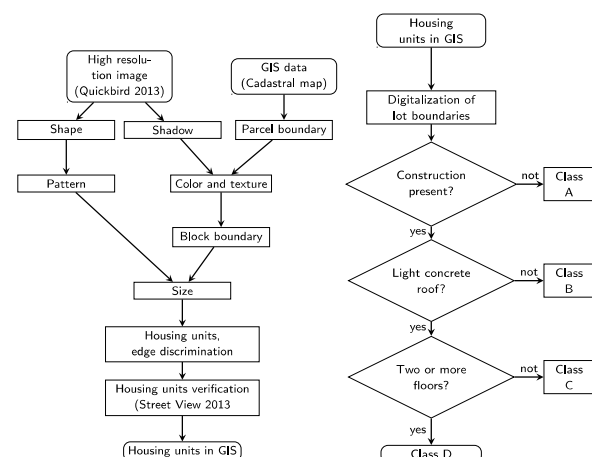Figure-3. Visual identification protocol

Figure-4. Categories of consolidation, from left to right, Top: A and B, Bottom: C and D


Class	Name	Description
A	Precarious	Provisional shelter or bare soil
B	Incipient	Basic structure with corrugated roofing
C	Consolidated one floor	First floor already finished and light concrete roofing
D	Consolidated two or more floors	Second floor finished and light concrete roofing

Table 1. Categories of consolidation

parcels: a) Location at urban fringe and belonging to a former agricultural parcel; b) Availability of high resolution imagery in the valley; c) presence of early stages of residential type of development at 2002. Sample is composed by three parcels (which are identified as parcels 1, 2, and 6) and their internal blocks (which are identified using letters following the number of the parcel they belong to, e.g. Block 1a). Sub-numbering is used to make it possible to identify blocks that belong the same parcel; number of sampled blocks sums 13 in total. Sampling methodology was performed using this hierarchical spatial structure composed by parcels, blocks and houses. Sampling blocks vary on number of houses and on composition and configuration of states. This heterogeneity allows to keep variety in the sample, and compel us to seek a general rule that could replicate each condition. Classification of house into clear states categories, uses the typology of consolidation states is taken from Tokeshi et al. (2005) who studied the self-building process in Lima [18]. Classification is implemented through a visual interpretation protocol (Fig. 3), which has been devised to permit an accurate identification and representation of the state at the level of individual houses using the imagery corresponding to the years 2002, 2006, 2009, 2011, 2013, and 2015. Selected sample present structural features that make possible to categorize the four states of house consolidation (Table 1 and Fig. 4). High-resolution imagery retrieved form Google Earth (Fig. 5) is used to identify visually informally developed parcels and to proceed to the classification of construction states of houses. A spatial data base is build using GIS, and maps are created at the scales of Parcel, Block and Houses. Spatial analysis of this observed pattern is going to be performed and will provide insights in the

Figure-5. Satellite images of Chillon valley (left) and Block 1a (right). Source: Google Earth
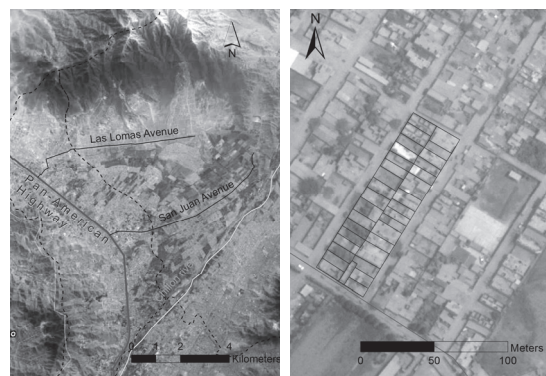
hypothesized effect of state of surrounding neighbors in the state of each house at a given time. Basic mapping unit is the house that contains construction state attribute and thus provide straightforward capabilities for spatial analysis of state transition.

(2) Description of state transition

Transition maps were build using information from choropleth maps of states; up to six types of transition have been identified at each stage: A to B, A to C, B to C, B to D, C to D (Fig. 6). In the first period the two most frequent are A to B and B to C. Studied blocks have similarities and differences in the trends of each of the four states. These trends can be summarized as a sharp decrease in A class, and a significant presence of class B at the first and second periods. The proportion of B class remains important in the blocks. Transitions in or to consolidated or rich states (C and D) vary in different ways, but the most common are B to C and C to D. These trends of change are also observed in the trajectories of state transition. As expected, state transition trajectories show a general trend of depletion of A state, and different patterns of increasing of states B, C and D. As time passes on a cumulative effect of D growth is observed 12 of the 13 presented blocks; the only block that does not show increasing trajectories of C and D states is block 2a; it further shows low transition rate in states A and B (poor states) as well.

Five characteristics of state transition process are identified: a) all non-consolidated houses that upgraded its state were surrounded by houses with similar or greater states, b) upgraded houses of consolidated and non-consolidated states form aggregated clusters of same state, c) initial state configurations have effect in the final configuration, d) upgraded houses at each period are allocated randomly, and e) access to major road influence on the rate of state transition, as the least accessible blocks (2a, 2b, 2c, 2e) are also the least developed. The systematic procedure of identifying the first characteristic starts by mapping two categories of change for the periods 2002-2009 and 2009-2015: houses that changed (variable) and houses that does not changed its state (invariable). Those houses that change its state were identified and the state of orthogonal neighbors were recorded using the corresponding state

Figure-6. Choropleth maps of states in block 1a (left to right: 2002, 2006, 2009, 2011, 2013 and 2015
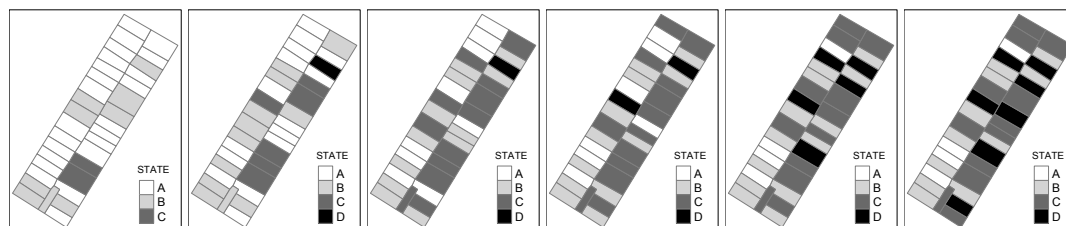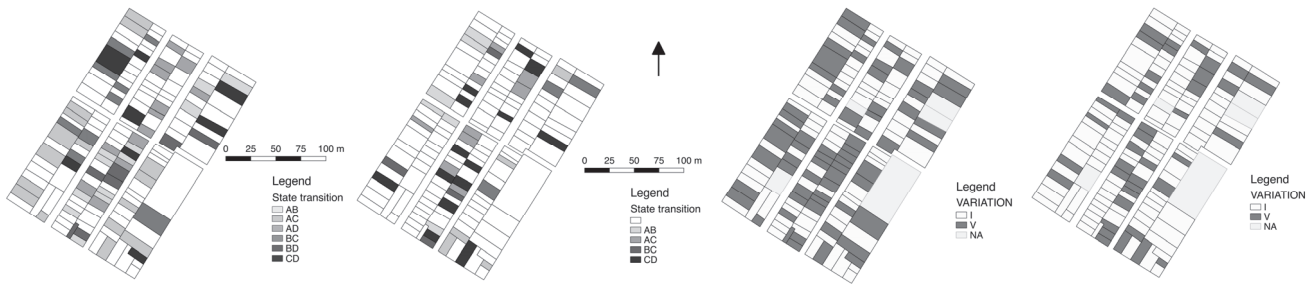
Figure-7. Transition maps of parcel 1 in periods 2002-2009 and 2009-2015 (left) and Variation maps in periods 2002-2009 and 2009-2015 (right), in the later maps non-residential uses are included as NA class.

map. This micro-scale analysis provides empirical based information to devise the fundamental algorithm for house state upgrading: all non-consolidated houses that upgraded its state were surrounded by houses with similar or greater states. Described conditions identifies and describes the spatial conditions of house upgrade that address the issue of changing spatial composition and configuration that drives upgrade, the likelihood of state transition and the spatial configuration in the block now of this transition considering the state of the neighbors of a house at the upgrade moment. The second mentioned characteristic is identified visually in the status maps, which groups lower (A and B) and higher (C and D) states within two categories. Third characteristic is observed when comparing choropleth maps of years 2002 and 2015. Fourth characteristic is determined by observing transition and variation maps, if constant and regular variation in upper states is present, maps show no regular allocation of allocation of consolidated houses (C and D). Finally, the fifth characteristic is taken from the analysis of relative access to major roads that each of the sampled parcels and its component blocks have. Although all five characteristics are considered in the design of the model, the first, fourth and fifth are the drivers and therefore used directly in the design of the algorithm (the updating rule); second and third characteristics are effects from the other three mentioned factors.

(3) Devising the rule of the model from empirical data

Transition process along the years can be tracked through the maps and graphics (Fig. 7). These spatial and temporal data are used to generate rules of transition, investigating how the latest states are related to early conditions, which refers to the concept of path dependence that is characteristic in complex systems. Rule-based behavior is chosen to represent individual performance of house-represented agents; dynamic interactions arises when rule is repeated along time steps; implementation of the rule modifies also the spatial configuration of house states into the lattice that represent the block into the CA model. Devised rule is implemented into the CA model of consolidation; the entire algorithm comprises both the parameters that provide the spatial setting and the rule that allow the operation; consequently, initial conditions are given by setting first the block size and initial composition and configuration in the block, then replicating the rule onto randomly allocated cells into lattice, considering changing composition and configuration of states along time-steps. Block size, the relative composition of A, B, or C states, and the registered distribution into the block is equal to the data identified at the initial stage (2002). Once initial settings are defined, algorithm selects randomly one of the cells into the lattice to start the rule operation; it is necessarily to remark that as seen in empirical information, any state can be selected to evaluate its likelihood to upgrade. Only one cell is selected randomly and evaluated against the rule at each time-step. Rule is performed by observing the states in the selected neighbors; here is where updating rule is applied: if average state of neighbors is greater than that on the target cell, then cell upgrades to the next state, if not another cell is selected randomly and thus rule is performed on it again until the stop conditions. Another observation focus on the effect of access in the state transition rate; poor accessible parcels and blocks have less number of consolidated houses compared to those with good access; thus, this characteristic is going to be included into the algorithm as a conditional parameter to be applied when simulating different conditions of access. Analysis is focusing only in those houses that have upgraded in the reference periods: 2002 - 2009 and 2009 to 2015.

(4) Cellular automata model of consolidation

1) Purpose. Is to simulate the consolidation process of houses into a block and to study how initial state composition and relative accessibility have effects on state transition process. The model intends to explain the foundation of the spatial pattern and state transition at block scale, in terms of both internal and external factors (local interaction and access respectively).

2) Entities, state variables, and scales. Main entities in the model are the houses, which are changing states from least developed state towards finished or consolidated houses. Agents are non-mobile cells; these cells are organized into a clearly defined space: the block. Two initial parameters that define the model are block size and initial composition of state values into the block. The environment entity is the block, which groups the houses. House neighbors have effect on its state. The state variable is the construction state at any point of time for each house. The regular and clearly delimited space of a housing block is represented as a lattice-like space composed by cells that corresponds to houses in different states of consolidation. The spatial configuration of the block comprises houses arranged in two rows of houses, which is typical in the observed housing blocks in Chillon valley. Initial settings consider defining block size (the number of houses in the block) and the initial composition and configuration of states into the block.

3) Process overview and scheduling. A non-synchronized CA model is implemented in the software Netlogo (Wilensky 1999). Houses represented by cells evaluate its state one per time-step using a simple updating rule that considers the average value of states in surrounding cells prior upgrading. Spatial relationships and effects among the cells are represented. Rule assumes a sensing property where "agents" read the state attribute of surrounding fellows. Reading means detecting state values in their neighborhood.

4) Initialization and calibration. The space of the block forms the environment in which houses interact and are influenced. "The use of hierarchies is another way to calibrate a model; often the sequence of

Figure-8. Actual trajectories of state transition in blocks 1a, 2a, and 6a
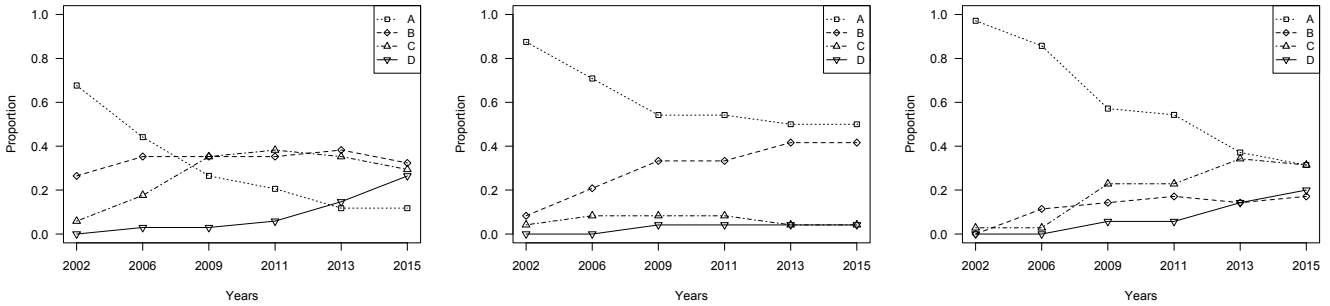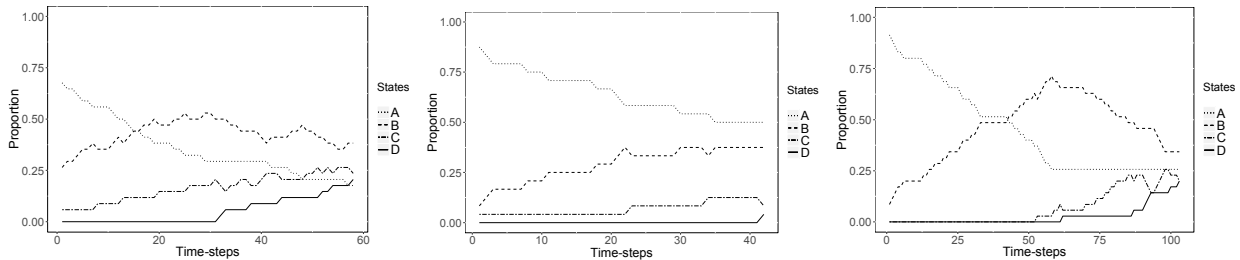

Figure-9. Simulated trajectories of state transition in blocks 1a, 2a, and 6a

urbanization adheres to a particular formal sequence"[19]; a clear transition hierarchy exist in the real world; consolidation sequence follows a unidirectional progress across states from A to D. Two additional and interchangeable conditional transition rules are also considered as stopping rule to cease state transition, which use the number of houses on state D. Calibration was performed by running the model 100 times to test the combined effect of the two-access related conditional rules. The model is designed to simulate consolidation in residential houses, therefore it is simulated validated using those blocks whose buildings present only residential use; thus, selected blocks were: 1a, 1c, 2a, 2b, 6a, and 6c.

3. Simulation of consolidation

Actual process is represented by maps and state transition graphics; simulated outcomes are presented similarly (Fig. 9). In first selection of 13 blocks, house composition shows they are in different stages of evolution along consolidation; this variety in states drive emergence of heterogeneous spatial patterns at the scale of the block. The Invariant-Variant method [20] is used to describe "how well the model performs, the situations or instances in which it does not perform well, and the cases in which it is relatively unlikely to predict well because of either path-dependence or stochastic uncertainty". Initial four states are grouped in two categories or status: a) Developed/Consolidated (C and D) and b) non-developed/non-consolidated (A and B). This method has the advantage to identify and measure the accuracy considering zones that changes (variant) and does not change (invariant) across multiple runs of the model, giving a dynamic perspective on the performance of the model. Validation is applied in six representative blocks, whose size, and initial

state composition and spatial configuration was reproduced by the model. Simulation runs three times each and resulting spatial configuration was categorized per the two categories. Average correct indicator compares spatial matching of simulated cells with those patterns retrieved from empirical data for the year 2015. Validation method comprises comparison of number of correct house status allocation by the model, against allocation by random selection, which uses the ratio C/R; the accuracy in the whole model is based on the spatial outcomes where the invariant region of consolidated (ID) and non-consolidated status (IU) are considered, which is measured across multiple runs in the invariant zone that is always occupied by those houses that have either been correctly (IC) or incorrectly (II) allocated by the model; accuracy considering only the variant region is implemented by comparing correct allocations in variable region to those allocations produced randomly, using the ratio VD/VRD (Table 2). The effect of random allocation component in the updating rule along the simulated process of consolidation is still noticeable, given the presence of a zone where different runs did not coincide (variant zones) and allocation of developed cells was not entirely determined by aggregation of similar categories. Values of C/R indicator in most blocks except 6c are greater than one, which means that accuracy is greater than 50% and therefore the model reproduces location of consolidated houses better than simple random allocation. Model performs well in invariant region and its spatial accuracy allows it to be generalizable for prediction of consolidation. In the other hand, most values of VC/VRD shows that in the variant region allocation is more due to random effect, and thus the rule is not representing well the process in the variant region. Specifically, these low values in the variant accuracy may describe that the model have the tendency of replicate different ways

	Block 1a			Block 1c			Block 2a			Block 2b			Block 6a			Block 6c		
Indicator	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
Average Correct (%)	42	47	42	30	70	40	50	50	50	44	22	22	50	53	56	90	60	50
ID size	5	5	5	1	1	1	1	1	1	1	1	1	6	6	6	6	6	6
IU size	12	12	12	2	2	2	21	21	21	5	5	5	7	7	7	1	1	1
C/R	1.20	1.56	3.36	2.00	0.75	1.18	23.98	6.00	6.00	1.12	1.14	2.25	1.09	1.09	1.23	0.95	0.83	0.53
VC/VRD	0.51	0.84	3.19	1.68	0.64	0.98	0	0	0	0.82	0.61	1.36	0.94	0.93	1.09	0.92	0.58	0.31

Table 2. Indicators of accuracy of outcomes from blocks 1a, 1c, 2a, 2b, 6a and 6c

of consolidation in blocks 1a, 1c, 2b, 6a and 6b (block 2a has null values in this specific indicator). This indicator on the accuracy of the variant zone complement previous indicator; accuracy in the variant region show many blocks with low values compared with those from random allocation. Good C/R values indicates that the accuracy of the model relies mostly on matching the same invariant region many times and most of them correctly. Thus, using the status composition and configuration at the block, model have predictive accuracy, but does not show process accuracy as matches in the variant region are less predictive than random allocation, therefore in this variant region path dependence is based on random consolidation rather than a specific spatial influence by neighbors. Interpreted results show first that model predictability is limited when allocating non-consolidated blocks in most of the simulated cases. Moreover, as the selected blocks present a range of different initial conditions and sizes, these stochastic uncertainties in consolidation process (found in most blocks except block 6c), may indicate that this type of stochastic process is characteristic of house consolidation in the context of informal development. Block 6c present both predictive and process accuracy and is an uncommon case among studied blocks. Thus, the model is generalizable as it contains an acceptable level of accuracy on the predictive capabilities in the other blocks; lack of accuracy in the variant region where the different run allocations show that path dependence is less dependent from the influence component of the updating rule than from its stochastic component of the rule.

4. Conclusions

The consolidation model fits to specific case of informal urban growth at Chillon, with a general state transition rule that transcend the specifics of a location using simple updating rule that acknowledges neighbors influence; it also is applicable under conditions of decentralized self-building decision-making, and non-restrictive building regulations at house scale. Updating rule provides both a generalizable and explicit update mechanism and resembles the stochastic nature observed in the actual process. A spatial-temporal experiment using this model showed that this explicit mechanism controls state transition trajectories and spatial patterns under the effect of initial conditions. Patchiness in a distinctive feature of spatial configuration at block scale in intermediate stages towards consolidation. Limitations of the model are highlighted by Invariant-Variant indicators, which suggest that the degree of predictability in the model was affected first, by the small number of invariant houses and second, by random allocation component of update rule. Predictability is improved when four state outcomes are aggregated into two classes (Consolidated and Non-consolidated). Good accuracy values of C/R indicator in all blocks except 6c show that model performs well in invariant region and its spatial accuracy allows it to be generalizable for prediction of house consolidation (residential use only). In the other hand values of VC/VRD show in most of the results that in the variable region allocation is more due to randomness effect, and thus the rule is not representing well the process in the variant region. Model has located consolidated houses with better precision in the invariant zone than the variant zone where random allocation has prevailed; it could be due to two possibilities: a) the influence rule on adjacent neighbors does not reflect accurately the actual process, or b) influence effect within the space of the block have a range beyond adjacent neighbors. Therefore, development in the variant region may depend on other causes than only spatial influence from adjacent neighbors; further work oriented to disentangle these possibilities should include a more explicit representation of the dynamics of the social dimension within the space of

the block. Current research may provide insights for planners in public and private institutions who faces informal development by allowing them to have a dynamic picture of the process and more accurate and bottom level accounts on the conditions of consolidation in houses and blocks; scenarios based on this model provide a possibility for tailored interventions at micro-scale.

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